Utilitarianism without Utility: A Missed Opportunity in Alfred Marshall's Theory of Market Choice

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In his early economic writings[1] Alfred Marshall analyzed the basic market choice—whether to accept or turn down a proposed transaction—in terms of an internal conflict between opposing desires: that of acquiring property or other rights, on the one hand, and that of keeping hold of what is required in exchange, on the other. Transactions in which the forces of the conflicting desires happen to balance constitute the so-called margin of indecision. Indeed, in modern societies practically all transactions are settled by means of money transfers. Thus, the desire for money can be taken to be a common ingredient of all these internal conflicts. In marginal transactions, the desire to keep or acquire the amount of money transferred offsets the *net* resultant force of all the other desires involved. Consequently, if it were possible to measure its force, the desire for money would provide a common measure of the net force of all the various desires, the satisfaction of which is attained through marginal transactions. Lacking such a measurement, we can at least take the amount of money transferred, which is measurable, as a rough, indirect indicator of force. Economics

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1. The relevant texts for the present discussion are the manuscript essays "Value" and "Money" (early 1870s), published in Marshall 1975, vol. 1; "Mr Mill's Theory of Value" (1876) and "The Present Position of Economics" (1885), republished in Marshall 1925.

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Typically, these will include *subjective* features, such as the person's information, expectations, and wealth, and his normal pattern of life, including the transactions in which he is usually involved;[5] and *objective* features, such as which transactions can normally be carried out in the market.

Let T stand for the set of all conceivable transactions. Those that are *feasible* at state s, that is, those that are available on the market at s, form the feasible set at s. Of course, if t is feasible at s and $t' \gtrsim_P t$, t' should also be feasible; t being available, however, no sensible economic agent would accept t', unless inadvertently or constrained by previous commitments. The relevant part of the feasible set is, therefore, its lower boundary with respect to the $\gtrsim_P$ relation. This will be indicated as $T(s)$. A transaction t belongs to $T(s)$ if and only if it is feasible at s, and no transaction t' such that $t' \lesssim_P t$ is feasible.

$T(s)$ conveys all the relevant information about prices and market structures at s. For example, if a commodity X can be bought in any amount $x \geq 0$ at unit price p, $T(s)$ will include all the transactions of the form $t = \langle x, px \rangle$. Quantity rationing, nonlinear pricing, and all sorts of market peculiarities can likewise be represented.

Given a transaction $\hat{t} = \langle X, \hat{P} \rangle$, Marshall's definition of the subjective money value P_m that a person attributes to X and, consequently, of the rent R attached to $\hat{t}$, at a given state s, involves a procedure of elicitation which ideally requires the person to step out of s and to imagine what he/she would do if faced with a whole sequence of counterfactual states. In order to clarify the procedure, it will help to define a "line of transactions with object X,"

$$L(X) := \{t : t = \langle X, P \rangle, P \in Z\},$$

where Z is the set of integers.[6] Thus, the given $\hat{t}$ is a point on a line $L(X)$, and we ask the person to determine which $t_m \in L(X)$ is the marginal copy of $\hat{t}$.

For our request to make sense, a marginal transaction must naturally exist in $L(X)$, and it must be unique. As this is far from obvious, let us specify it formally as a distinct assumption:

5. "Normal" and "usual" are typical Marshallian qualifications that serve to emphasize the deliberate vagueness of the definition of the context of choice. In accordance with these qualifications, occasional transactions should not be part of the description of the current state until they have become habits.

6. As P stands for money payments, it is natural to take it as a discrete variable (money has a smallest unit). My argument would not substantially change if P were treated as a real or a rational variable.

A1 At each state s, any line of transactions $L(X)$ contains one and only one marginal transaction.

A1 implies that, at each s, $L(X)$ is divided into two connected subsets: an "inferior" subset containing all the acceptable transactions, and a "superior" one containing all the ones that are rejected. Since we are treating P as a discrete variable (see footnote 6), there is room for ambiguity as to which of the two "bordering" transactions should play the role of marginal transaction: the one with the largest P for which t is accepted, or the one with the smallest P for which t is rejected. We will follow the former convention.

In order to find out his/her own t_m, the subject of the elicitation experiment is instructed to consider each $t \in L(X)$ and to decide whether to accept or reject it in the hypothesis that t is (1) feasible and (2) an all-or-nothing transaction, that is, that there is no other feasible transaction that makes it possible to trade different amounts of the same commodity or service represented by X. Since $L(X)$ at most contains only one transaction that belongs to $T(s)$, and there is no guarantee that this is an all-or-nothing transaction, for each $t \in L(X)$ with the possible exception of one, the subject has to think of an imaginary state that differs from s as much as is necessary to meet the conditions 1 and 2.

These differences will also be assumed to be the only ones as far as the *objective* features of the two states, actual and imaginary, are concerned. In the next section we shall see that we also need to consider *subjective* differences with regard to the person's plan of expenditure.

Once t_m has been identified, P_m is uniquely determined, and so are the rents associated with all the $t \in L(X)$, including the given $\hat{t}$. From the way the experiment is designed, P_m depends only on X and on the current state s, $P_m = P_m(X, s)$. Consequently, for each $t = \langle X, P \rangle$ in $L(X)$ the rent R depends on X, s, and P, or, for short, $R = R(t, s)$. Of course, if both t and t' belong to $L(X)$ and $\delta = P' - P$, it follows that $R(t', s) - R(t, s) = -\delta$.

2. The Margin as a Representation of the "Importance" of Money

Although the concepts introduced so far apply both to demand and supply behavior, from now on the argument will refer only to the case of consumer demand. All the transactions considered, therefore, must be understood to be purchases of commodities or services with a positive P.

Let $A(s)$ stand for the person's plan of expenditure at s, that is, that part of the description of the state s that refers to the transactions included in the person's normal pattern of activity. Obviously, in order to carry it out, a transaction must be feasible, $A(s) \subseteq T(s)$. We have to define the conditions under which $A(s)$ is *consistent* with the monetary evaluations made by the person at s.

For each X, the intersection $L(X) \cap T(s)$ contains the transactions with object X that are feasible at s. If it is not empty, it contains only one transaction, which we indicate with $t_s = \langle X, P_s \rangle$. Three cases are possible, $P_s \gtreqless P_m(X, s)$ or, equivalently, $R(t_s, s) \lesseqgtr 0$. In the first case, the definitions in section 1 imply that the person rejects t_s in the (possibly counterfactual) hypothesis that it is an all-or-nothing transaction. If t_s is not all-or-nothing at the actual state s, however, what should the decision be? Here, we can apply a basic principle of choice theory (known as the Chernoff condition, from Chernoff 1954) according to which the choice from a given menu is not affected by the elimination of nonchosen items from the menu. An equivalent formulation of it is as follows:

A2 If a choice is rejected from a menu S, it continues to be rejected from all the menus that include S.

Removing the all-or-nothing hypothesis amounts to enlarging the person's field of choice. If t_s is rejected on an all-or-nothing basis, A2 entails that it should be rejected also in the actual state, where all-or-nothing does not apply. Thus, A2 and the way in which the elicitation procedure is designed imply the proposition

P1 If $t \in T(s)$ and $R(t, s) < 0$, then $t \notin A(s)$,

from which, of course, it follows that

P2 If $t \in A(s)$, then $R(t, s) \geq 0$.

It can easily be seen, however, that the reverse of P2 does not generally hold, that is, $t \in T(s)$ and $R(t, s) \geq 0$ are not sufficient for t to be carried out at s. Indeed, under the definitions in section 1, $R(t, s) \geq 0$ implies that, in the all-or-nothing hypothesis, the person accepts t. If all-or-nothing does not hold in the actual state s, however, it is possible that t is rejected in favor of some other transaction in which a different amount of the same commodity is purchased.

Here, our attention must be turned to the case of decomposable transactions. As t is a vector, it can be decomposed into a sum of vectors that have additive components. Thus, suppose that the nonmoney object of $t_1, \dots t_n$ is the same as the object of t, and that the vector sum $t = \sum_h t_h$ for $h = 1, \dots n$

holds. Provided $t_h \in T(s)$ for all h, saying that $t \in A(s)$ is the same as saying that $t_h \in A(s)$ for all h.[7] So, if t is a feasible transaction with nonnegative rent but is excluded from the expenditure plan at s, it cannot be part of a "larger" transaction included in the same expenditure plan. If t is rejected in favor of some t' dealing with the same commodity, the amount purchased with t' must be smaller than the one purchased with t. This leads to the following conclusion:

P3 If $t = \langle X, P \rangle \in T(s)$, $R(t, s) \geq 0$ and $t \notin A(s)$, then $A(s)$ must contain (at least) one transaction $t' = \langle X', P' \rangle$ with $X' < X$.

It follows from P3 that, if a commodity X is completely absent from the plan of expenditure at s, and transactions $t_x = \langle x, P_x \rangle \in T(s)$ are feasible for some amount x of the commodity, then $R(t_x, s) < 0$ for all feasible x.

P1–P3 characterize the states at which the expenditure plans are *consistent* with the subjective monetary evaluations. Therefore, a consistent $A(s)$ will generally be made up of two parts: the one that we may call the *actual margin* at s,

$$M(s) = \{t \in A(s) : R(t, s) = 0\},$$

and the *inframarginal* part $I(s) = A(s) / M(s)$. In particular cases, one of the two may be empty.

Let us now go back to the counterfactual mental experiments by means of which rents are assessed. While Marshall is never very clear as to what is covered by his ceteris paribus clauses, an ideal elicitation procedure should state explicitly that, at each hypothetical state, only consistent plans of expenditure should be considered.[8] The consequence is that, given a transaction $\hat{t} = \langle X, \hat{P} \rangle \in A(\hat{s})$, the subject of the experiment, in looking for his/her marginal copy $\hat{t}_m$ in $L(X)$, has to consider two imaginary states for each $t = \langle X, P \rangle$ with $P \geq \hat{P}$, say s_1 and s_2: the two must satisfy conditions 1 and 2 of section 1 above plus a new condition 3, according to which

7. Of course, this does not extend to all the additive decompositions of t, but only to those whose addenda belong to $T(s)$. In the case of a commodity X sold at fixed unit price p, $t = \langle x, px \rangle \in A(s)$ is equivalent to $t' = \langle x', px' \rangle \in A(s)$ for all the amounts $x' \leq x$ currently traded.

8. It may be objected that this requirement is not realistic. Normally, in elicitation experiments, people are asked to assess how much they would pay for something without greatly pondering how to restructure their plan of expenditure in order to accommodate the new item. Such an objection, however, only casts doubts on the reliability of this type of experiment: should the imaginary situation become true, the subject of the experiment would be forced to adapt his plan of expenditure to the circumstances, and there is no guarantee that the hypothetical evaluation expressed in the experiment would be confirmed.

$t \in A(s_1) / A(s_2)$, and both $A(s_1)$ and $A(s_2)$ are consistent. This means that s_1 is the imaginary state at which $P \geq \hat{P}$ is spent for the purchase of X, while at s_2 neither X nor any smaller amount of the same commodity (from condition 2) is bought, and P is allocated among other transactions with non-negative rents (from the consistency of $A(s_2)$). When $t >_P \hat{t}$, $A(s_1)$ necessarily differs from $A(\hat{s})$ because the difference $P - \hat{P}$ must be funded by cutting down on other transactions; on the other hand, $A(s_2)$ differs from $A(\hat{s})$ for all $t \geqq_P \hat{t}$ because, at all s_2, X is excluded and $\hat{P}$ is reallocated in a consistent way among other transactions.

It must result from the experiment that, for all t up to $\hat{t}_m$, expenditure plan $A(s_1)$ is preferred to $A(s_2)$, and for $t >_P \hat{t}_m$ the reverse. Let s_m indicate the imaginary state s_1 associated with $\hat{t}_m$, that is, what we may call the "marginal state" of $\hat{t}$. At s_m, keeping X is preferred to giving it up—with no possibility of purchasing smaller amounts of the same commodity—in exchange for what can be obtained with the best allocation of P_m. However, at the state corresponding to the immediately successive t in $L(X)$, the preference is reversed and the best allocation of $P_m + 1$ is preferred to keeping X. This confirms the interpretation of P_m as the *money equivalent* of the net benefit expected of X as evaluated at s_m, or, inversely, of the net benefit expected of X as a *real equivalent* of an amount of money P_m available at s_m.

Commodities traded through intramarginal transactions have their money equivalents at marginal states s_m that generally differ from the actual state $\hat{s}$. On the contrary, when a transaction is marginal the difference between its own marginal state and $\hat{s}$ becomes immaterial: in this case, the only difference lies in the all-or-nothing hypothesis which, on the strength of A2, is practically irrelevant. The actual margin at $\hat{s}$ thus provides a list of amounts of money—the Ps of all the $t = \langle X, P \rangle$ included in $M(\hat{s})$—for each of which a real equivalent, identified by means of an evaluation based on the actual state, is indicated. The margin is found to be a qualitative indicator, a sort of "peephole" on the "real meaning" or "real importance" of money for a given person at a given state.

The assumptions on which this reconstruction of the Marshallian framework is based are not strong enough to guarantee that the margin is always non-empty. It is conceivable that, at a certain state s, all the realized transactions are inframarginal, $A(s) = I(s)$. In such cases, the notion of the real importance of money becomes purely virtual: real equivalents still exist, but they must refer to imaginary marginal states, possibly all different tarian approach. While the marginal utility of money can only increase, decrease, or stay put with changes of state, the margin intended as a set of transactions has many more ways of changing. Some of these have inevitably been lost, as we can see from a few examples.

Suppose that the current state changes from s to s', for example, as a consequence of an increase in the price of some commodities. In order to make room for the greater expense in the latter, the subject gives up some transactions that previously belonged to the margin. It may not be necessary to dismiss all the transactions in the margin, however. Thus, it may be the case that $M(s) \neq M(s')$, but $M(s) \cap M(s') \neq \varnothing$; some transactions marginal at s remain marginal at s', and therefore serve to provide a common indicator of the importance of money in both states. Now, what if we try to tell the same story in terms of the marginal utility of money?

As the marginal utility of money coincides with the utility obtained through expenditure in marginal transactions, and as there are transactions in $M(s) \cap M(s')$, one version of the story might conclude that the marginal utility of money is the same at s as at s'. Some transactions, however, have left the margin during the switch from s to s', while others may have entered it. Suppose $t' = \langle Y, Q \rangle$ to be a transaction in $M(s) / M(s')$ and, to put the problem into sharper focus, suppose also that Y is an indivisible commodity. Then, from P3 of section 2, t' must have a negative rent at s'; hence, $P_m(Y,s') < P_m(Y, s) = Q$. With the marginal utility of money unchanged, it must be supposed that the utility of Y has decreased during the shift from s to s'. If there is no reason why this should have happened, however, the only possible conclusion is that, far from remaining constant, the marginal utility of money must have increased. But this takes us back to the transactions in $M(s) \cap M(s')$. Let $t = \langle X, P \rangle$ be one of these; then $P_m(X, s') = P_m(X,s) = P$. An increased marginal utility of money implies that the utility of X must have increased exactly in the same proportion from s to s'. Again, what if we cannot find any reason for this?

The conclusion as to what has happened to the marginal utility of money is, to say the least, unclear. The complication, however, seems mainly due to the overly precise language of the marginal utility story. In the qualitative language of the money transfer approach, $P_m(Y, s')$ is smaller than $P_m(Y, s)$, and t' is consequently dismissed. This is simply because at s' the person can no longer afford Y, but the change is not palpable enough to represent a change in the importance of money for the margin of individuals representative of given classes; (3) if needs of the same importance are satisfied at the margin for one, inside the margin for another, or if needs satisfied at the margin of one are more important than those at the margin of another, to conclude that money is more important for the former than for the latter. Ultimately, the basis for Marshall's comparative judgments of welfare is the importance of money as revealed by a sampling of the contents of the margin typical of each social group, combined with a conventional agreement as to standards of priority in the satisfaction of human needs. This position seems nearer to the contemporary concern for social indicators than to the utilitarian tradition of the old welfare economics.

4. Determinateness of Marginal Demand Prices without the Constancy of the Marginal Utility of Money

The last step in this reconstruction of Marshall's "monetary utilitarianism" is a reexamination of his concept of marginal demand price and of the demand function that he derived from it.

The virtual elicitation experiment underlying the marginal demand price can be described as follows. Take a commodity X which, at the current state s, is traded in discrete amounts 1, 2 . . . x . . . For each amount $x \geq 1$, the person in the experiment is asked to locate his marginal transaction in $L(x)$ and in $L(x-1)$, so as to define

$$\delta(x, s) = P_m(x, s) - P_m(x-1, s).$$

The difference $\delta(x, s)$ can be taken to indicate the money equivalent (evaluated at s) of the net benefit expected from a shift from an imaginary marginal state in which the person is constrained to purchase either the quantity $x-1$ or none of X to one in which the quantity constraint has been relaxed to x. To put it briefly, the δs represent the subjective monetary values of relaxing hypothetical quantity constraints on X by one unit.

Marshall's hypotheses concerning the marginal demand price, which I identify from now on with δ, are two: first, the hypothesis that δ is monotonically decreasing with respect to x; second, that the comparison between each $\delta(x,s)$ and the market unit price of X is decisive for determining the optimal amount of X in the expenditure plan $A(s)$. Assume that p is the unit price of X at s: under the first hypothesis, x is optimal at s if p is not greater than $\delta(x,s)$—a one-unit relaxation of the quantity constraint $x - 1$ is at least worth p—and greater than $\delta(x + 1,s)$—further relaxation of the constraint is not worthwhile.

So far, this analysis is consistent with the general drift of Marshall's monetary approach to market choices. We can think of $\delta(x,s) - p$ as being the rent attached to a fictional transaction consisting of the "purchase" of a one-unit relaxation of an $x - 1$ constraint on X against payment of p. When $\delta(x,s) - p \geq 0$, the inclusion of this transaction in the expenditure plan at s is a straightforward consequence of P3 in section 2. From P1 and $\delta(x + 1,s) - p < 0$ it also follows that a similar transaction concerning the x constraint, although feasible, should not be included in $A(s)$. Thus, at a state s in which X can be bought in any amount for a unit price p, a plan of expenditure containing all the transactions $t_h = \langle h,ph \rangle$ up to $h = x$ is consistent if and only if $p \leq \delta(h,s)$ for $h \leq x$ and $p > \delta(h,s)$ for $h > x$.

It can be noted at this point that, from the definition of the δ function, we have

$$P_m(x,s) = \sum_{h=1}^{x} \delta(h,s).$$

Consequently, the rent attached to a transaction $t = \langle x, px \rangle$ can be expressed as

$$R(t,s) = \sum_{h=1}^{x} [\delta(h,s) - p] = \sum_{h=1}^{x} R(t_h^*,s), \tag{1}$$

where t_h^* stands for the fictional transaction exchanging a one-unit relaxation of the h constraint for the payment of p. This formula corresponds to Marshall's graphic representation of consumer rent by means of the "triangle" between the demand curve and the price line—with an important proviso that will be discussed shortly.[12]

One last remark concerning the fictional transactions t_h^*: when the equality $\delta(x,s) - p = 0$ holds, x being the amount bought at s, the transaction t_x^* belongs to the margin $M(s)$. The margin, therefore, may be richer than it appears from a mere inventory of the "visible" transactions in $A(s)$, as it can contain marginal parts of intramarginal transactions. This remark indicates an additional reason (see section 2) why, in Marshall's theoretical case of quantities treated as continuous variables, the margin should never be empty.

All this analysis, however, does not support the next step taken by Marshall at this point, namely that of interpreting the list of the marginal

12. This formula also indicates that the optimal x is the one that maximizes the rent with respect to all the available amounts of X.

questions such as "how much of anything people would buy at prices very different from those which they are accustomed to pay for it" (1961, 1:133). Implicitly, he suggested that, provided conjectures were kept within a limited neighborhood of the observed situation, the difference between the δ function and the demand function might be held to be negligible for all practical purposes.

Indeed, the assumption that Marshall needed in order to bear out this claim is seen to be weaker than the two hypotheses mentioned above. What is required is that, for any given state s, a set of states be found, let us call it $K(s)$, such that each $s' \in K(s)$ differs from s with respect to the price of X, be it p', and the plan of expenditure $A(s')$, but has $\delta(h,s') = \delta(h,s)$ for at least two values of h, $h = x'$ and $h = x' + 1$, with x' being the amount purchased at s'. Within such a class of states, the list of the marginal demand prices evaluated at s may well play the role of a "local" demand function. The assumption is weak also in that it implies neither symmetry nor transitivity: s' may belong to $K(s)$ without s belonging to $K(s')$; and s'' may belong to $K(s')$, with $s' \in K(s)$, but not to $K(s)$. In other words, the δ function may change with the state: what is important is that, at each state, at least a part of the demand function be approximated by some values of the δ function at that state.

A sufficient condition for this assumption to hold is that, at each $s' \in K(s)$, the subjective evaluations $P_m(h,s')$ remain unchanged over the relevant range of values of h. Although more restrictive than necessary, this condition implies less than the constancy of the marginal utility of money and separable utilities. It may be enough that the change in price between s and s' is not large enough to bring about either a shift of the whole margin—that is, a change in the importance of money—or considerable variations in the amounts of substitutes and complements present in the plan of expenditure, or both.

References

Aldrich, J. 1996. The Course of Marshall's Theorizing about Demand. *HOPE* 28:171–217.

Chernoff, H. 1954. Rational Selection of Decision Functions. *Econometrica* 22:422–43.

Dardi, M. 2006. Demand. In *The Elgar Companion to Alfred Marshall*, edited by T. Raffaelli, G. Becattini, and M. Dardi, 293–300. Cheltenham: Edward Elgar.

Friedman, M. 1949. The Marshallian Demand Curve. *Journal of Political Economy* 57:463–95.